OPTICAL FIBERS

1

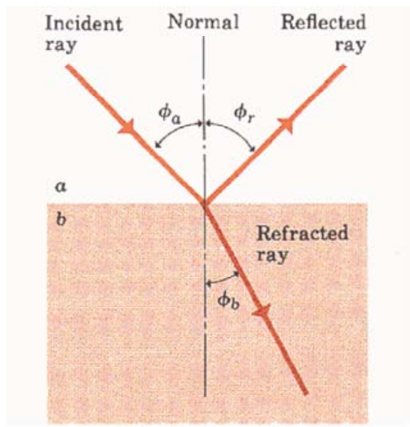
WE FOCUSE ON

- Optical fiber dispersion
- Transmission speed limited due to dispersion
- Losses in OF
- Non-linear effects in fiber optics

2

Optical Ray

Refraction and Reflection



$$\phi_a = \phi_b$$

$$n_a \sin\phi_a = n_b \sin\phi_b$$

Let's consider that we investigate the directions of the incident, reflected and refracted rays of monochromatic light. We fill find the following results illustrated by Fig. :

1. The incident, reflected and refracted beams and the normal to the surface, all lie in the same plane.
2. The angle of reflection ϕr is equal to the angle of incidence ϕa (ϕr=ϕa).
3. For a given pair of substances, *a* and *b*, on opposite sides of the surface of separation, the ratio of the sine of the angle ϕa (between the beam in substance a and the normal) and the sine of angle ϕb (between the beam in substance b and the normal) is a constant (sin ϕa/sinϕb=constant).

Refraction and Reflection

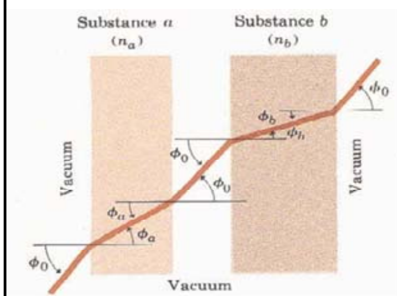


Fig. 1-3-1

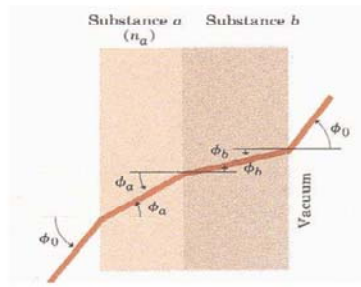
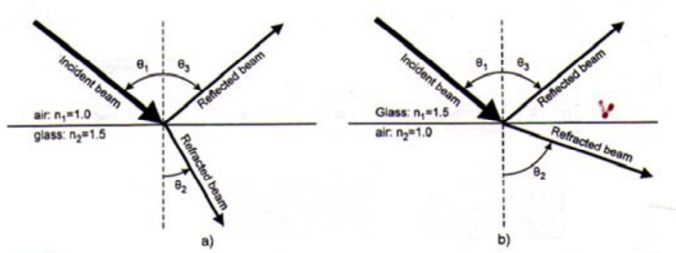


Fig. 1-3-2

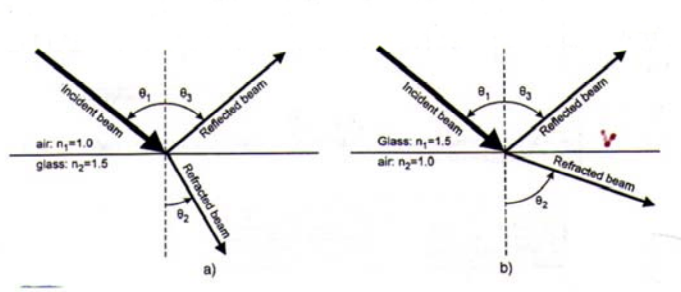
The angles in figure 1-3-1 are independent of the thickness and space between the two plates and are the same when the space shrinks to nothing, as in figure 1-3-2.

Problema

- Fie n1=1, θ1=30°, n2=1.5.
- Care este unghiul θ2 pentru cele doua cazuri din fig1.



Solutie



$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1}{1.5}\sin 30° = 0.333$$

$$\theta_2 = \arcsin(0.333) = 19.5°$$

$$\sin\theta_2 = \frac{n_1}{n_2}\sin\theta_1 = \frac{1.5}{1}\sin 30° = 0.75$$

$$\theta_2 = \arcsin(0.75) = 48.6°$$

Total Internal Relection

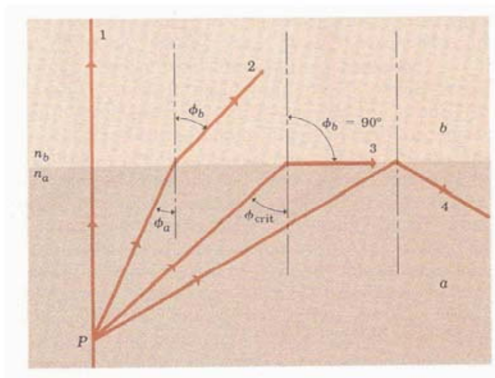


Fig. 1-4-1

$$\sin\phi_{crit} = \frac{n_b}{n_a}$$

Figure I-4-1 shows a number of rays diverging from a point source *P* in a medium *a* of index *n*a and striking the surface of a second medium *b* of index *n*b, where *n*a>*n*b. The angle of incidence for which the refracted ray emerges tangent to the surface is called critical angle ϕcrit. At this angle ϕb=90^o and Snell's law becomes *n*asinϕa=*n*b, since sin90^o=1. We then have with ϕa=ϕcrit.

Optical Fiber

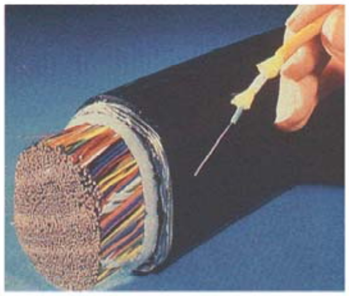


Fig. 1-4-3a

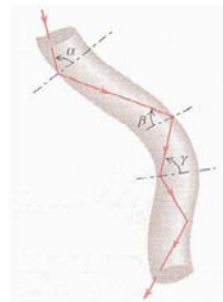


Fig. 1-4-3b

$$\sin \phi_{crit} = \frac{n_b}{n_a}$$

A very important application of total internal reflection is the fiberoptic cable shown in figure I-4-3 (a). Figure I-4-3 (b) shows the working principle of the cable. When a beam of light enters at one end of the transparent fiber, the light is totally reflected internally and is trapped within the rod.

Dispertion

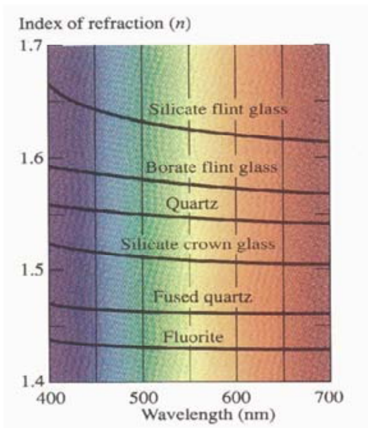


Figure I-5-1

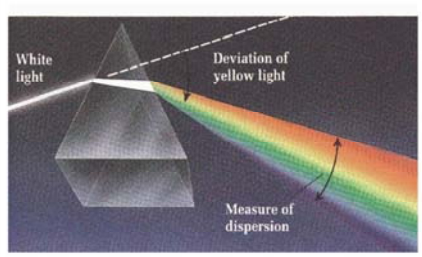
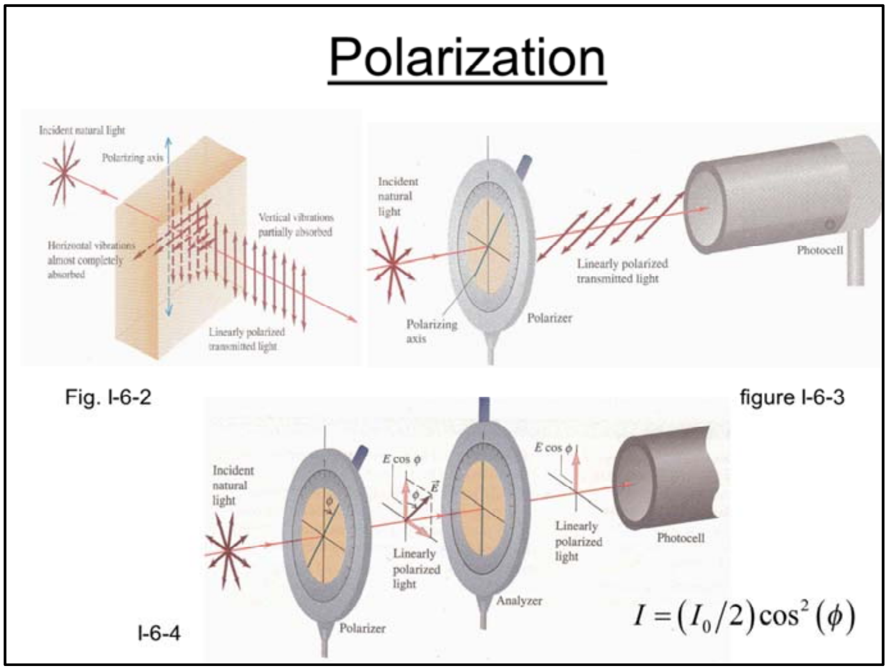


Figure I-5-2

Ordinarily, white light is a superposition of waves with wavelengths extending throughout the visible spectrum. The speed of light in vacuum is the same for all wavelengths, but the speed in a material substance is different for different wavelengths. Therefore the index of refraction of a material depends on the wavelength. The dependence of the index of refraction on the wavelength is called **dispersion**. Figure I-5-1 shows the variation of the refractive index with the wavelength for different optical materials. The value of *n* usually decreases with increasing wavelength and
thus increases with increasing frequency. Light of longer wavelength usually has greater speed in a material than light of shorter wavelength.

Figure I-5-2 shows the ray of white light incident on a prism. The deviation (change of direction) produced by the prism increases with increasing the refractive index and frequency (i.e., the energy).



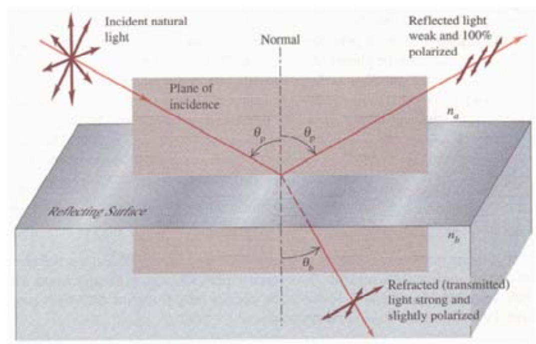
Linearly polarized light can be produced from unpolarized light with the aid of certain materials. One commercially available material goes under the name of Polaroid. As shown in figure I-6-2, such materials allow only the component of the electric field along one direction to pass through, while absorbing the field component perpendicular to this direction.

Light from ordinary sources is not polarized. The “antennas” that radiate light waves are the molecules that makes up the sources. The waves emitted by any one molecule may be linearly polarized. However, any actual light source contains a tremendous number of molecules with random orientations, so the light emitted is a random mixture of waves that are linearly polarized in all-possible directions.

In figure I-6-3 unpolarized light is incident on a polarizer. The blue line represents the polarizing axis. The ***E*** vectors of the incident wave exhibit random directions. The polarizer transmits only the components of ***E*** parallel to the polarizing axis. The intensity of the transmitted light is exactly half of the incident unpolarized light, no matter how the polarizing axis is oriented. Here’s why: We can resolve the ***E*** field of the incident wave into a component parallel to the polarizing axis and a component perpendicular to it. Because the incident light is a random mixture of all states of polarization, these two components are, on average, equal. The (ideal) polarizer transmits only the component that is parallel to the polarizing axis, so half of the incident intensity (*I*0/2) is transmitted.

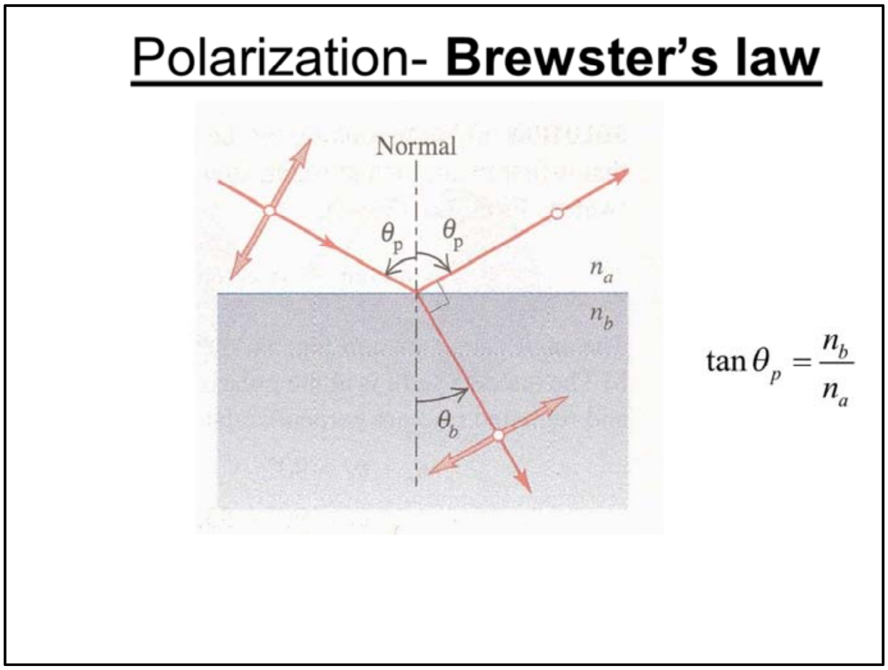
What happens when the linearly polarized light emerging from a polarizer passes through a second polarizer, as shown in figure I-6-4? To find the transmitted intensity at intermediate values of the angle ϕ, we bear in mind that the intensity of an electromagnetic wave is proportional to the square of the amplitude of the wave. The ratio of the transmitted to incident amplitude is cosϕ, so the ratio of transmitted to incident intensity is cos2ϕ. Thus, the intensity of the light transmitted through the analyzer is Where, *I*0 is the maximum light intensity at ϕ=0. Equation (I-6-1) is called **Malus's law**.

Polarization



A further possibility to create either partially or totally polarized light is by reflection. In figure I-6-5, unpolarized light is incident on a reflectin surface between two transparent optical materials. The plane containing the incident and reflected rays and the normal to the surface is called the **plane of incidence**.

At one particular angle of incidence, called the polarizing angle θ_p, only the light for which the ***E*** vector is perpendicular to the plane of incidence is reflected. The reflected light is therefore linearly polarized perpendicular to the plane of incidence (i.e., parallel to the reflecting surface).
In 1812, Sir David Brewster noticed that when the angle of incidence is equal to the polarizing angle θ_p, the reflected and refracted ray are perpendicular to each other. The situation is shown in figure I-6-6. In this case $\theta_b=90^o-\theta_p$. Using equation (I-3-1), we find $\sin\theta_p/\sin(90^o-\theta_p)=\sin\theta_p/\cos\theta_p=n_b/n_a$ and finally



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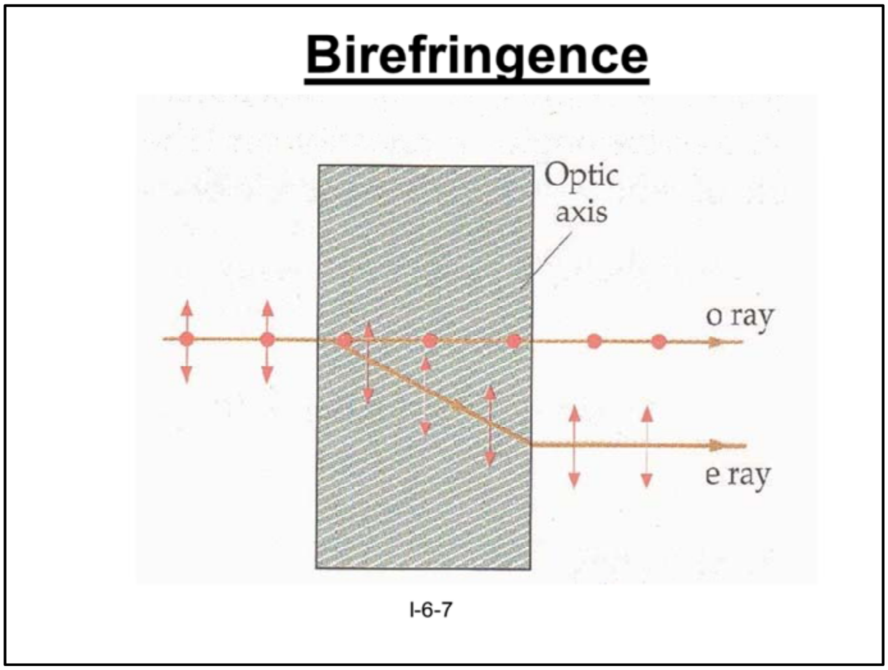
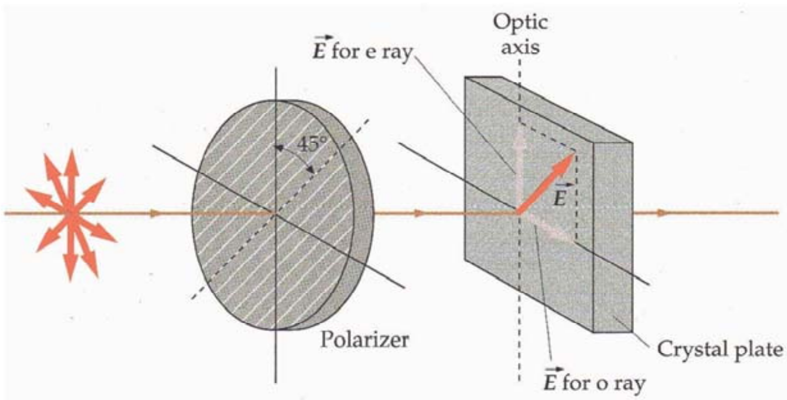


Light and other electromagnetic radiation can also have circular or elliptical polarization, i.e., the ***E*** describes a circular or elliptical rotation. In this context **polarization by birefringence** is important. Birefringence occurs in calcite and other noncubic materials (hence also in various semiconductors) and some stressed plastics and cellophane. Most materials are **isotropic**, that is, the speed of light passing through the material is the same in all directions. Because of their atomic structure, birefringent materials are **anisotropic**. The speed of light depends on its direction of propagation through the material. When a light ray is incident on such materials it may be separated into two rays called the **ordinary** and **extraordinary** ray. There is one particular direction in a birefringent material in which both rays propagate with the same speed. This direction is called the **optic axis** of the material. However, when light is incident at an angle to the optic axis, as shown in figure I-6-7, the rays travel in different directions and emerge separated in space.

Birefringence



I-6-8

If light is incident on a birefringent plate perpendicular to its crystal face and perpendicular to the optic axis, the two rays travel in the same direction but different speeds. The rays emerge with a phase difference that depends on the thickness of the plate and on the wavelength of the incident light. In a **quarter-wave plate**, the thickness is such that there is a 90° phase difference between the waves of a particular wavelength when they emerge. In a half-wave plate, the rays emerge with a phase difference of 180°.

Suppose that the incident light is linearly polarized such that $\boldsymbol{E}$ is 45° to the optic axis, as illustrated in figure I-6-8. The ordinary and extraordinary rays start out in phase and have equal amplitudes. With a quarter-wave plate, they emerge with a phase difference of 90°, so the resultant components of $\boldsymbol{E}$ are $\boldsymbol{E}x=\boldsymbol{E}_0\sin\omega t$ and $\boldsymbol{E}y=\boldsymbol{E}_0\sin(\omega t+90^o)=\boldsymbol{E}_0\cos\omega t$ ($\omega=2\pi\nu$ is the angular frequency and t represents the time). The electric field vector thus rotates in a circle and the wave is circularly polarized.

Birefringence

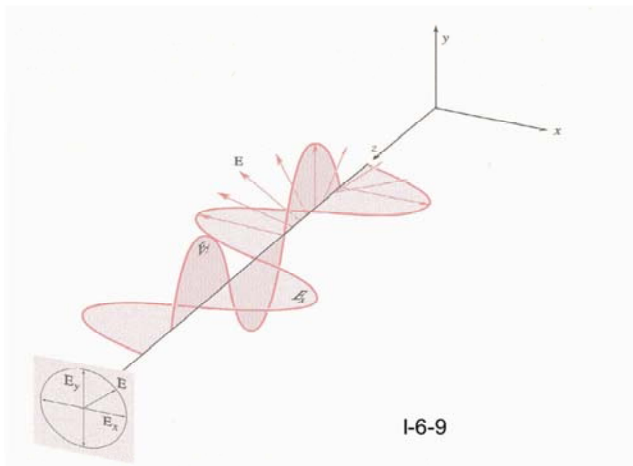
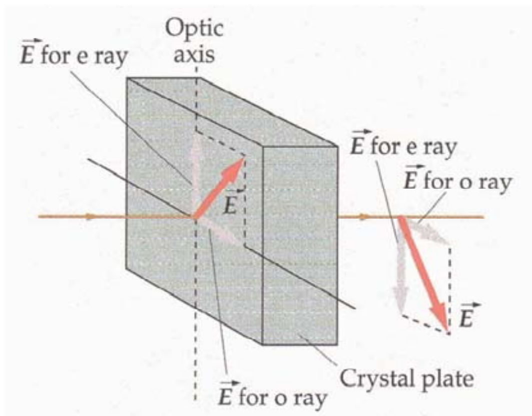


Figure I-6-9 shows the propagation of circular polarized light. If the advancing wave revolves clockwise (looking toward the source), then it’s said to be right-circularly polarized; if counterclockwise, it’s left-circularly polarized. The magnitude of ***E*** remains constant while revolving once around with every advance of one wavelength. As the wave advances, the electric
field vector ***E*** rotates clockwise once around per wavelength. The magnitude of ***E*** is constant.

Birefringence

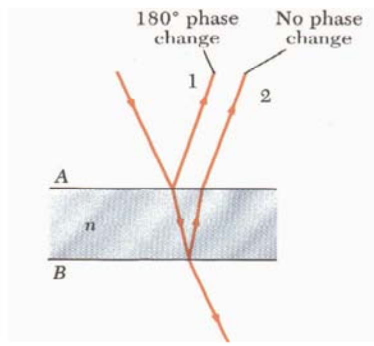


I-6-10

With a half-wave plate, the wave emerge with a phase difference of 180o, so the resultant electric field is linearly polarized with components ***E***x=***E***0sinω*t* and ***E***y=***E***0sin(ω*t*+180o)=-***E***0sinω*t*. Hence, the direction of the wave polarization is rotated by 90o relative to that of the incident wave, as shown in figure I-6-10.

If the phase difference between the two components of ***E*** is something other than a quarter wavelength, or if the two component wave have different amplitudes, the resulting wave is elliptically polarized.

Thin films



$$2nd = (m + 1/2)\lambda, \quad (m = 0,1,2,\dots) \quad (1)$$

$$2nd = m\lambda, \quad (m = 0,1,2,\dots) \quad (2)$$

$$d = \frac{\lambda_1 \lambda_2}{2n(\lambda_1 - \lambda_2)} \quad (3)$$

We consider now a thin film of uniform thickness *d* and index of refraction *n* shown in figure I-8-2. To determine whether the reflected light rays interfere constructively or destructively, we must note the following fact: A wave traveling in a medium of low refractive index (air) undergoes a 180o phase change upon reflection from a medium of higher refractive index. There is no phase change in the reflected wave if it reflects from a medium of lower refractive index.

Ray 1 is reflected from the upper surface *A* undergoes a phase change of 180o with respect to the incident wave. On the other hand, ray 2, which is reflected from the lower surface *B* undergoes no phase change with respect to the incident wave. Therefore, ray 1 is 180o out of phase with ray 2 corresponding to path difference of λn/2. However, we must consider that ray 2 travels an extra distance equal to 2*d* before the waves recombine. Hence, if 2*d*=λn/2=λ/(2*n*) the phase difference between both rays is 360o and the waves recombine in phase and constructive interference takes palace. In general, the condition for constructive interference is expressed as (1) and for destructive interference we have (2).

Thin films are of considerable importance for the formation semiconductor devices. Almost all optoelectronic devices are composed of the combination of various thin films. Concerning research and development, by means of optical spectroscopy not only the optical or optoelectronic features of semiconductors are investigated but also other features as the film thickness. Hence, in any cases optical characterization methods accompany the manufacturing steps of electronic and optoelectronic devices. For optical thickness measurements, equations (1) and (2) can be used to determine the film thickness. According to equation (2), the fringe of order m lies at $\lambda 1$ and that of order $(m+1)$ at $\lambda 2$. Hence, we have $m\lambda 1=(m+1)\lambda 2$ so that $m=\lambda 2/(\lambda 1-\lambda 2)$. With (2) we find, $nd=\lambda 1\lambda 2/(\lambda 1-\lambda 2)$ and the thickness of the film is (3), where $\lambda 1$ and $\lambda 2$ is the wavelength of two adjacent maxima or minima in the spectrum.

Thin films - exemplu

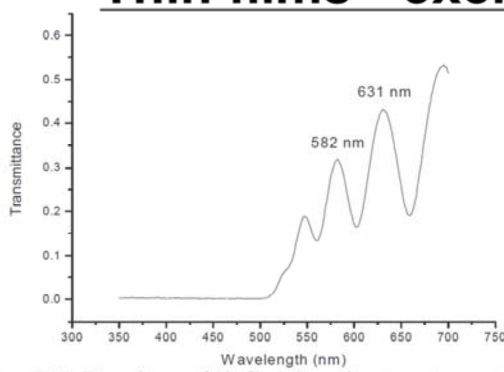


Figure I-8-3: Transmittance of thin film CdS on glass at room temperature.

$$d=\frac{\lambda_1\lambda_2}{2n\left(\lambda_1-\lambda_2\right)}=\frac{\left(582\times10^{-9}m\right)\left(631\times10^{-9}m\right)}{5\times\left(49\times10^{-9}m\right)}=1.5\mu m \quad (3)$$

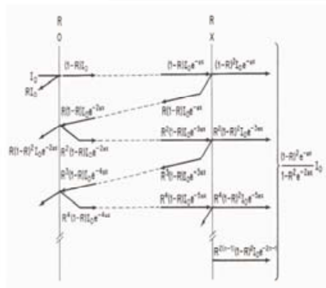
Example

Figure I-8-3 shows the transmission spectrum of a thin CdS film on glass. The transmittance starts at the band-gap of the material (≈500 nm) and pronounced fringes at 582 and 631 nm are observed.

Solution

The thin film CdS exclusively causes the fringes in figure I-8-3. The glass substrate does not influence the interference effect. Hence, we insert the wavelengths of the two indicated maxima and n_{CdS}=2.5 in equation (3) and get the thickness of the film,

Thin films



I-8-4

$$Tr = \frac{(1-R)^2 e^{-\alpha x}}{1-R^2 e^{-2\alpha x}} \quad (4)$$

$$\mathrm{Re} = R\left\{1+\frac{(1-R)^2 e^{-2\alpha x}}{1-R^2 e^{-2\alpha x}}\right\} (5)$$

$$\mathrm{Re} = R\left\{1+(1-R)^2 e^{-2\alpha x}\right\} \quad (6)$$

$$Tr = (1-R)^2 e^{-\alpha x} \ (7)$$

The calculation of the transmitted and reflected intensities of thin films requires the consideration of the internal reflections. Figure I-8-4 shows the concept. *I*0 is the intensity of the incident beam, *R* is the reflection coefficient of the surface and backface, α is the absorption coefficient and *x* the thickness of the film. The transmitted and reflected intensities are summed up with a geometrical series delivering the following results for the transmittance and reflectance, (4) and (5). For many applications, the formulas (6) and (7) are accurate enough.

Thin films

Example:
Calculate the transmittance of the film with a thickness of 1 μm, an absorption coefficient of 100 cm-1 and an reflection coefficient of 0.2.

Solution:
Tr=(1-0.2)^2exp(-100 cm-1×10-4 cm)≈(1-0.2)2=0.64.

We see, in case of effective absorption the reflection of the surface determines the transmission features of the film.

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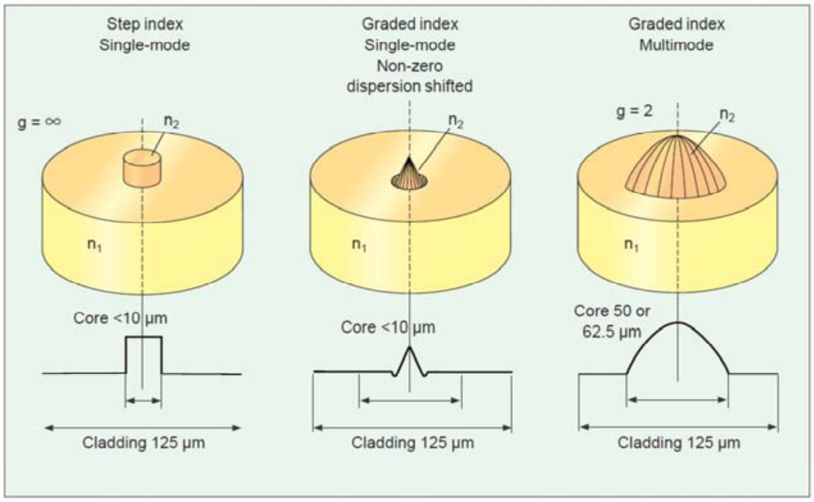
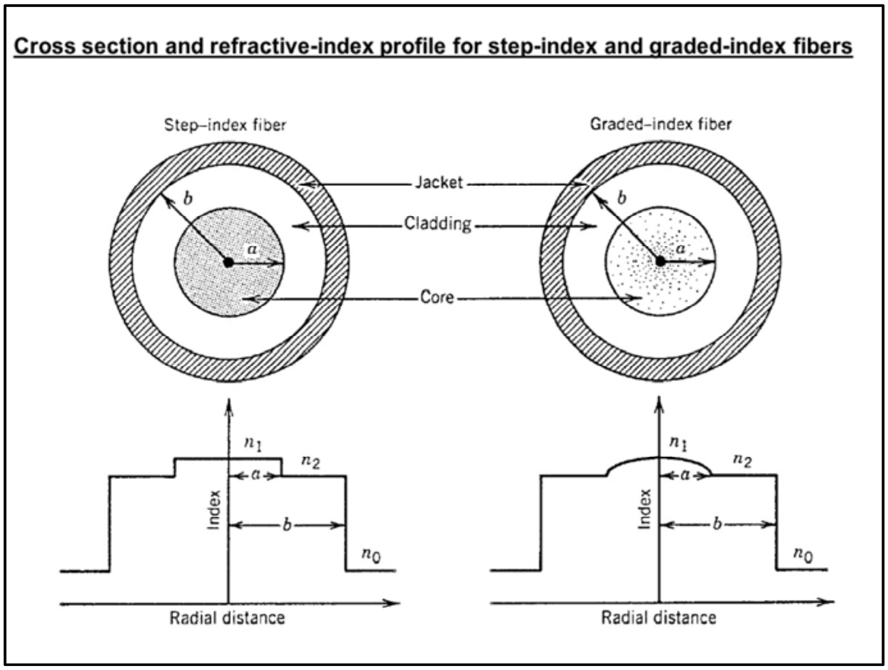
Cross section and refractive-index profile for step-index and graded-index fibers



Graphic representation of three different types of how the refractive index change in the core of an optical fiber.



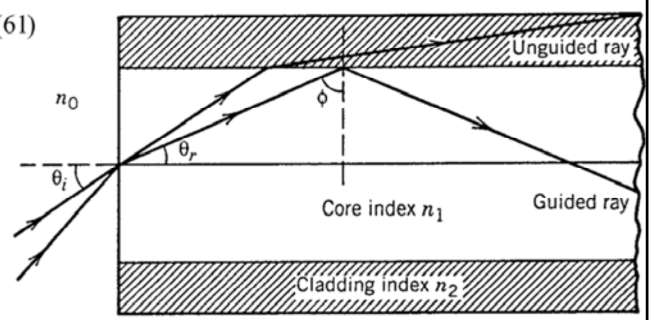
In its simplest form an optical fiber consists of a cylindrical core of silica glass surrounded by a cladding whose refractive index is lower than that of the core. Because of an abrupt index change at the core–cladding interface, such fibers are called *step-index fibers*. In a different type of fiber, known as *graded-index fiber*, the refractive index decreases gradually inside the core. Considerable insight in the guiding properties of optical fibers can be gained by using a ray picture based on geometrical optics. The geometrical-optics description, although approximate, is valid when the core radius *a* is much larger than the light wavelength λ . When the two become comparable, it is necessary to use the wave-propagation theory.

Step-index fiber

$$n_0 \sin\theta_i = n_1 \sin\theta_r \quad (61)$$

$$\sin\phi \le \frac{n_2}{n_1} \quad (62)$$

$$\sin\phi_c = \frac{n_2}{n_1} \quad (63)$$



$$n_0 \sin\theta_{ic} = n_1 \sin(90° - \phi_c) =$$
$$= n_1 \cos(\phi_c) = \sqrt{n_1^2 - n_2^2} = NA \quad (64)$$

$$n_1 \approx n_2$$
$$NA = n_1\sqrt{2\Delta}, \quad \Delta = \frac{n_1 - n_2}{n_1} \quad (65)$$

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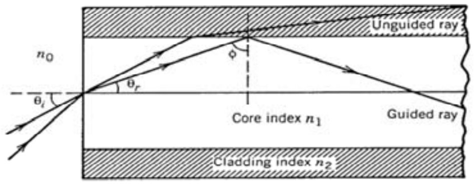
Consider the geometry of Figure, where a ray making an angle θ *i* with the fiber axis is incident at the core center. Because of refraction at the fiber–air interface, the ray bends toward the normal. The angle θ *r* of the refracted ray is given by (61), where *n*1 and *n*0 are the refractive indices of the fiber core and air, respectively. The refracted ray hits the core–cladding interface and is refracted again.
Refraction is only possible for angles that satisfy the relationship (62). For angles greater than the critical angle ϕc, the relation (63), where *n*2 is the cladding index, the ray is totally reflected. Since such reflections occur throughout the fiber length, all rays with ϕ >ϕ*c* remain confined to the fiber core. This is the basic mechanism behind light confinement in optical fibers.
One can use Eqs. (61) and (62) to find the maximum angle that the incident ray should make with the fiber axis to remain confined inside the core. Noting that θ*r* =π /2−ϕ*c* for such a ray and substituting it in Eq. (61), we obtain (64).
In analogy with lenses, *n*0 sinθ*i* is known as the *numerical aperture* (NA) of the fiber. It represents the light-gathering capacity of an optical fiber. For *n* 1~*n*2, the NA can be approximated by (65), where Δ is the fractional index change at the core–cladding interface. Clearly, Δ should be made as large as possible in order to couple maximum light into the fiber. However, such fibers are not useful for the purpose of optical communications because of a phenomenon known as multipath dispersion or *modal dispersion.*

Intermodal dispersion for a step-index fiber



$$BL < \frac{n_2}{n_1^2}\frac{c}{\Delta} \quad (69)$$

$$\Delta T = \frac{n_1}{c}\left(\frac{L}{\sin\phi_c} - L\right) = \frac{L}{c}\frac{n_1^2}{n_2}\Delta \quad (66)$$

$$\Delta T < T_B = \frac{1}{B} \quad (67)$$

$$\frac{L}{c}\frac{n_1^2}{n_2}\Delta < \frac{1}{B} \quad (68)$$

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Multimodal dispersion can be understood by the fact that the various core propagation modes travel on different lengths of roads. As a result, these modes (rays) reach the end of the fiber at different times, even if they have all traveled at the same speed. As a result, an impulse will be larger as a result of these different road lengths.

We can estimate the pulse widening, considering the smallest length and the longest length. The shortest path appears for θi = 0, and has a length equal to that of the fiber, say L. The longest path appears for θi given by the equation (64), and it has the length L / sin (ϕc). Considering that the propagation velocity is the same for both cases, c / n1, the time difference is given by the relationship (66).

We can link ΔT to the capacity of the information transport of the fiber B, measured by the bit rate. It is intuitive that ΔT should be less than the time allocated to a bit, TB=1?B (see ec.67). So, using the relationship (66), it results (68), respectively (69).

The relationship (69) is a rough estimate of the fundamental limitation of step-index fiber.

Exercise

$$BL < \frac{n_2}{n_1^2}\frac{c}{\Delta}$$

<u>Fibre without cladding</u>: n1=1.5 si n2=1.

BL<0.4 (Mb/s)-km.

<u>Fibre with cladding</u> has Δ < 0.01.

For example, for Δ = 2*10^(3) we have BL < 100 (Mb/s)*km

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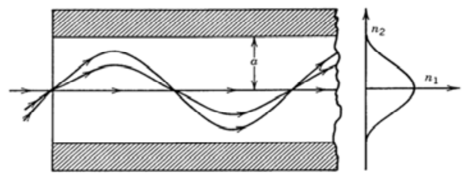
As an illustration, consider an unclad glass fiber with $n1 = 1.5$ and $n2 = 1$. The bit rate–distance product of such a fiber is limited to quite small values since BL<0.4 (Mb/s)*km. θ_i
Considerable improvement occurs for cladded fibers with a small index step. Most fibers for communication applications are designed with Δ < 0.01.
As an example, BL < 100 (Mb/s)*km for Δ = 2×10−3.
Such fibers can communicate data at a bit rate of 10 Mb/s over distances up to 10 km and may be suitable for some local-area networks.

Graded-Index Fibers



$$n(\rho)=\begin{cases} n_1\left[1-\Delta(\rho/a)^{\alpha}\right]; & \rho < a \\ n_1(1-\Delta)=n_2; & \rho \ge a \end{cases} \quad (70)$$

27

The refractive index of the core in graded-index fibers is not constant but decreases gradually from its maximum value $n1$ at the core center to its minimum value $n2$ at the core–cladding interface. Most graded-index fibers are designed to have a nearly quadratic decrease and are analyzed by using α-profile, given by (70), where a is the core radius. The parameter α determines the index profile. A step-index profile is approached in the limit of large α. A *parabolic-index fiber* corresponds to α = 2.

It is easy to understand qualitatively why intermodal or multipath dispersion is reduced for graded-index fibers. Figure 2.3 shows schematically paths for three different rays. Similar to the case of step-index fibers, the path is longer for more oblique rays. However, the ray velocity changes along the path because of variations in the refractive index. More specifically, the ray propagating along the fiber axis takes the shortest path but travels most slowly as the index is largest along this path. Oblique rays have a large part of their path in a medium of lower refractive index, where they travel faster. It is therefore possible for all rays to arrive together at the fiber output by a suitable choice of the refractive-index profile.

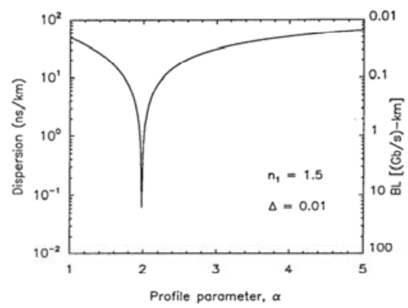
Intermodal dispersion in graded-index fibers

$$\alpha = 2(1-\Delta) \quad (73)$$

$$\frac{\Delta T}{L} = \frac{n_1}{8c}\Delta^2 \quad (74)$$

$$\Delta T < T_B = \frac{1}{B} \quad (75)$$

$$\boxed{BL < \frac{8c}{n_1 \Delta^2} \quad (76)}$$



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Intermodal dispersion in graded-index fibers has been studied extensively by using wave-propagation techniques. The quantity $\Delta T/L$, where ΔT is the maximum multipath delay in a fiber of length L, is found to vary considerably with α. Figure shows this variation of the dispersion for $n1$ = 1.5 and Δ = 0.01. The minimum dispersion occurs for $\alpha = 2(1-\Delta)$ and depends on Δ, as in (74). The limiting bit rate–distance product is obtained by using the criterion $\Delta T < 1/B$, *ec. (75)* and is given by (76).
The right scale in Fig. shows the *BL* product as a function of α. Graded-index fibers with a suitably optimized index profile can communicate data at a bit rate of 100 Mb/s over distances up to 100 km. The *BL* product of such fibers is improved by nearly three orders of magnitude over that of step-index fibers. Indeed, the first generation of lightwave systems used graded-index fibers. Further improvement is possible only by using single-mode fibers whose core radius is comparable to the light wavelength. Geometrical optics cannot be used for such fibers.

Exercitiu

O fibră cu indice gradat are apertura numerica 0.275 şi n1=1.487. Care este viteza de bit restricţionată de dispersia modală, pentru 1 km de fibră ?

Solutie

$$B_{GI} = \frac{8c}{n_1 \Delta^2} = \frac{8c}{n_1 \frac{NA^4}{4n_1^4}} = \left(32cn_1^3\right) \Big/ \left(L\left(NA\right)^4\right) =$$

$$= 5,5 \times 10^9 \text{ bit/s} = 5.5\,\text{Gbit/s}$$

Normalized frequency and modes

$$\boxed{V = \frac{\pi d}{\lambda} NA = kaNA}$$

$$N \approx \frac{V^2}{2} \frac{g}{g+2}$$

$$\boxed{N = V^2/2 \quad (4.7)}$$

$$\boxed{N = V^2/4 \quad (4.8)}$$

An important distinguishing feature of different types of optical fiber is the normalized frequency (V) and the number of modes (N). The following parameters affect the value of V:

D = core diameter in microni

NA = aperture numerica

λ = wavelength [microni]

k = wavelength's number

For a step index, g = ∞and the number of modes N is approximated by (4-7)

For a graded index fiber with g = 2 (normal parabolic graded index fiber), the number of modes N is approximated by (4-8)

Exercitiu

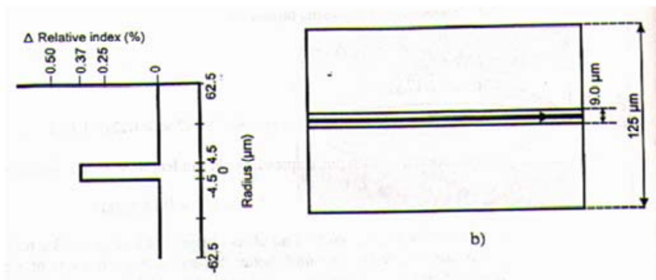
Calculaţi numărul de moduri dintr-o fibră cu indice gradat dacă diametrul miezului este 62.5µm, apertura numerică 0.275, iar lungimea de undă de funcţionare 1300nm.

Solutie

$$V = \frac{\pi dNA}{\lambda} = \frac{\left(3.14 \times 62.5 \times 10^{-6} \times 0.275\right)\text{m}}{1300 \times 10^{-9}\text{m}} = 41.5$$

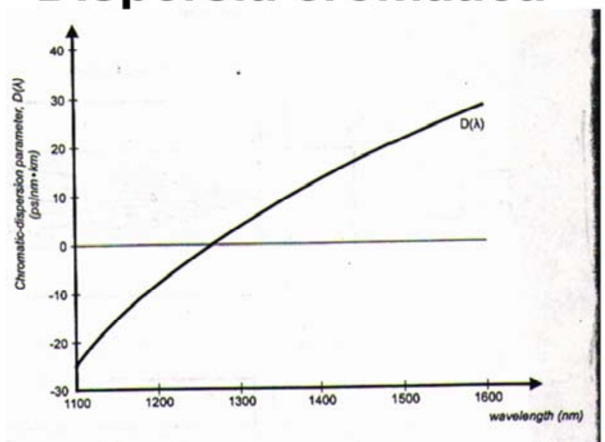
$$N = \frac{V^2}{4} = 431$$

Fibra monomod



$V \leq 2.405$

Dispersia cromatică



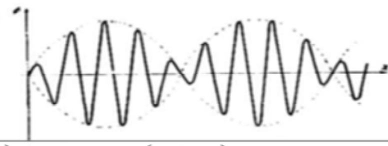
$$\Delta t_{crom} = D(\lambda) L \Delta\lambda$$

Dispersia totală

$$\Delta t_{total} = \sqrt{\left(\Delta t_{mod\,al}^2 + \Delta t_{crom}^2\right)}$$

Dispersion in monomod fiber
Group of Waves

$$\nu - \nu' = \Delta\nu$$
$$u - u' = \Delta u$$



$$\phi = a\sin 2\pi\nu\left(t - \frac{x}{u}\right) + a\sin 2\pi\nu'\left(t - \frac{x}{u'}\right) =$$
$$= 2a\sin\pi\left[t(\nu+\nu') - x\left(\frac{\nu}{u} + \frac{\nu'}{u'}\right)\right]\cos\pi\left[t(\nu-\nu') - x\left(\frac{\nu}{u} - \frac{\nu'}{u'}\right)\right]$$

$$\nu + \nu' \approx 2\nu \qquad \frac{\nu}{u} + \frac{\nu'}{u'} \approx 2\frac{\nu}{u} \qquad \frac{\nu}{u} - \frac{\nu'}{u'} \approx \Delta\left(\frac{\nu}{u}\right)$$

$$\boxed{\phi \approx 2a\sin 2\pi\nu\left(t - \frac{x}{u}\right)\cos\pi\left[t\Delta\nu - x\Delta\left(\frac{\nu}{u}\right)\right] = A\sin 2\pi\nu\left(t - \frac{x}{u}\right)}$$

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We have seen that intermodal dispersion in multimode fibers leads to a considerable increase in optical impulses (~ 10 ns / km). In the modal description of propagation, this phenomenon is due to the fact that each mode has a different group speed.
In mono-modal fibres, intermodal dispersion disappears because the energy is transported by a single mode.
However, the widening the impulse does not disappear. Group velocity associated with the fundamental mode of propagation is frequency dependent due to chromatic dispersion. Therefore, different pulse spectral components propagate at different group speeds, a phenomenon called intra-modal dispersion or group speed dispersion (GVD).
This dispersion has two components: material dispersion and guide dispersion.

Group of Waves- 2

$$t\Delta\nu - x\Delta\left(\frac{\nu}{u}\right) = 0$$

$$\boxed{\frac{1}{v_g} = \frac{t}{x} = \frac{\Delta\left(\frac{\nu}{u}\right)}{\Delta\nu} \xrightarrow{\Delta\nu\to 0} \frac{d}{d\nu}\left(\frac{\nu}{u}\right) = \frac{1}{2\pi}\frac{d\beta(\nu)}{d\nu}}$$

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Cromatic dispersion

$$v_g = \left(\frac{d\beta}{d\omega}\right)^{-1} \quad (75) \qquad v_g = \frac{c}{n_g}, \beta = \bar{n}k_0 = \bar{n}\frac{\omega}{c} \quad (76)$$

$$n_g = \bar{n} + \omega\frac{d\bar{n}}{d\omega} \quad (77)$$

$$\Delta T = \frac{dT}{d\omega}\Delta\omega = \frac{d}{d\omega}\left(\frac{L}{v_g}\right)\Delta\omega = L\frac{d}{d\omega}\left(\frac{1}{v_g}\right)\Delta\omega = L\frac{d}{d\omega}\left(\frac{d\beta}{d\omega}\right)\Delta\omega = L\frac{d^2\beta}{d\omega^2}\Delta\omega = L\beta_2\Delta\omega \quad (78)$$

$$\beta_2 = \frac{d^2\beta}{d\omega^2}$$

$$\Delta T = \frac{dT}{d\lambda}\Delta\lambda = \frac{d}{d\lambda}\left(\frac{L}{v_g}\right)\Delta\lambda = LD\Delta\lambda$$

$$D = \frac{d}{d\lambda}\left(\frac{1}{v_g}\right) = -\frac{2\pi c}{\lambda^2}\beta_2 \quad (79)$$

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Consider a fibre of length L. A spectral component of frequency would reach the fibre output after a time T = L / vg, where vg is the group speed defined by (75). Using (76) we obtain ng, the relationship (77).

Frequency dependence of group velocity leads to widening of the pulse due to different spectral components moving at different speeds. If Δω is the width of the pulse spectrum, the widening of the pulse in a length L of the fiber is given by the relationship (78). β2 is the chromatic or GVD dispersion.

In optical communication systems, Δω is determined by the range of wavelengths emitted by the optical source.

D is called **the dispersion parameter** and is measured in ps / (nm * km), see relationship (79).

Cromatic dispersion

$$BL|D|\Delta\lambda < 1 \quad (80)$$

$$\Delta T = \frac{dT}{d\lambda}\Delta\lambda = \frac{d}{d\lambda}\left(\frac{L}{v_g}\right)\Delta\lambda = LD\Delta\lambda$$

$$D = -\frac{2\pi c}{\lambda^2}\frac{d}{d\omega}\left(\frac{1}{v_g}\right) = -\frac{2\pi}{\lambda^2}\left(2\frac{d\bar{n}}{d\omega} + \omega\frac{d^2\bar{n}}{d\omega^2}\right) \quad (81)$$

$$D = D_M + D_W$$

$$D_M = -\frac{4\pi}{\lambda^2}\frac{d\bar{n}}{d\omega} = \text{dispersia de material}$$

$$D_W = -\frac{2\pi}{\lambda^2}\omega\frac{d^2\bar{n}}{d\omega^2} = \text{dispersia de ghid}$$

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The effect of dispersion is the reduction of the bit-rate B. This can be estimated by the condition (80).

For standard silica fibers, D is relatively small, around the 1.3 micron wave length. (D ~ 1ps / (km * nm).

For a laser diode, the spectral width Δλ is 2-4 nm. The BL product exceeds 100 (Gb / s) * km. Thus, 1.3 microns optical communications systems typically operate at 2 Gbps with a distance between repeaters of 40-50 km.

The BL product of monomodal fibers may exceed 1 (Tb / s) * km when the laser has a spectral width below 1 nm.

The dispersion parameter D may vary considerably when the operating wavelength differs considerably from 1.3 µm.

The D’s wavelength dependence is governed by the frequency dependence of the refractive index n_bar. From relation (79), D can be written in the form (81), where the sum of two terms: DM, material dispersion, and DW, the dispersion of the guide appear.

Material Dispersion

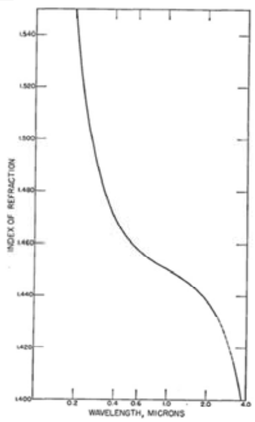
$$n^2(\omega) = 1 + \sum_{j=1}^{M} \frac{B_j \omega_j^2}{\omega_j^2 - \omega^2} \quad (82)$$

SILICA
B1 = 0.6961663, B2 = 0.4079426 , B3 = 0.8974794
Λ1 = 0.0684043 μm , λ2 = 0.1162414 μm , λ3 = 9.896161 μm

$$n_g = n + \omega \frac{dn}{d\omega} \quad (83) \qquad D_M = \frac{1}{c} \frac{dn_g}{d\lambda} \quad (84)$$

SILICA

$$D_M \approx 122\left(1 - \frac{\lambda_{ZD}}{\lambda}\right) \quad (85)$$



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The material dispersion occurs because the core refractive index changes with the optical frequency ω. The origin of the material dispersion is related to the characteristic resonant frequencies at which the material absorbs electromagnetic radiation. At frequencies far from environmental resonances, the refractive index n (ω) can be approximated by the Sellmeier (82) [I.H. Malitson, Interspecimen Comparison of the Refractive Index of Fused Silica, Journal of the Optical Society of America, 55 (10), 1205-1209, 1965), where ωj is the resonance frequency and Bj is the oscillation intensity. N can be n1 or n2, after the area of interest. The sum in Equation (82) is made at all resonance frequencies that contribute to the domain of interest. In the case of optical fibers, Bj and ωj are obtained empirically by fitting the dispersion curves measured at Equation (82) with M = 3.
The group index, ng, can be obtained with the relationship (83).

The dispersion of material, DM, is related to the slope of the group index through the relationship (84). For silica, derivatives are canceled to λZD = 1.276 μm. This wavelength is called zero-wavelength, λZD. The material dispersion is negative under this λZD and becomes positive over it.

In the wavelength range of 1.25 - 1.66 μm, the dispersion of material can be approximated empirically by the relationship (85).

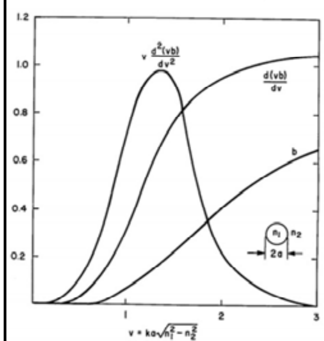
The zero dispersion wavelength also depends on the diameter of the core and the parameter D.

In domeniul de lungimi de unda 1.25 – 1.66 μm, dispersia de material poate fi aproximata empiric prin relatia (85).

Lungimea de unda de disperse zero depinde si de diametrul miezului si parametrul Δ.

Waveguide Dispersion

$$D_W = -\frac{2\pi}{\lambda^2}\omega\frac{d^2 \bar{n}}{d\omega^2} = -\frac{2\pi}{\lambda^2}\omega\frac{d^2}{d\omega^2}\left[n_2\left(1+b\left(V\right)\Delta\right)\right] \quad (86)$$



$$D_W = -\frac{2\pi\Delta}{\lambda^2}\left[\frac{n_{2g}^2}{n_2\omega}\frac{Vd^2\left(Vb\right)}{dV^2} + \frac{n_{2g}}{d\omega}\frac{d\left(Vb\right)}{dV}\right] \quad (87)$$

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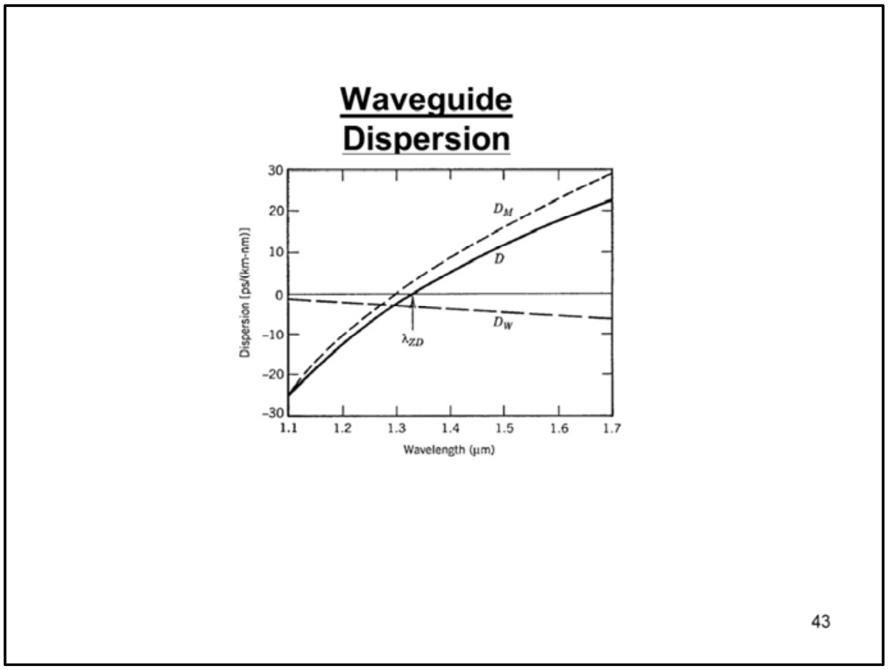
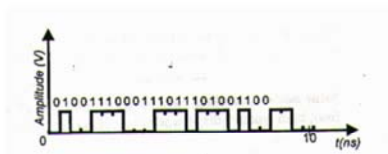
The contribution of the guide dispersion DW to the total dispersion is given by the relation (86), where b (V) is the normalized propagation constant. Using the relation (17) for V in (86) we get the relation (87). In figure [D. Gloge, Dispersion in Weakly Guiding Fibers, Applied Optics, 10 (11), 2442-2445, 1971] presents the modification with v of the derivatives of orders 1 and 2 of bV. Since both derivatives are positive, DW is negative across the range of interest: 0-1.6 µm.



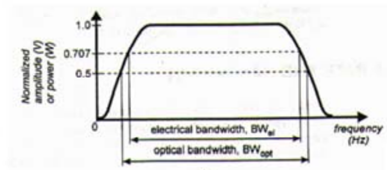
Since DW <0 and DM is negative for wavelengths shorter than λZD and positive for wavelengths greater than λZD, in the figure we notice that their sum D = DW + DM is zero at a wavelength shifted by 30-40 nm, so that for silicon, the zero value for D is 1.31 µm.

DW also reduces DM from 1.3 to 1.6 µm, which is of interest to optical communication systems.

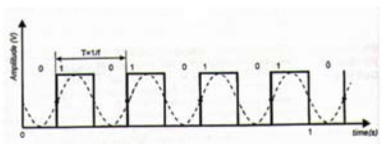
Bit velocity and bandwidth



a)



b)



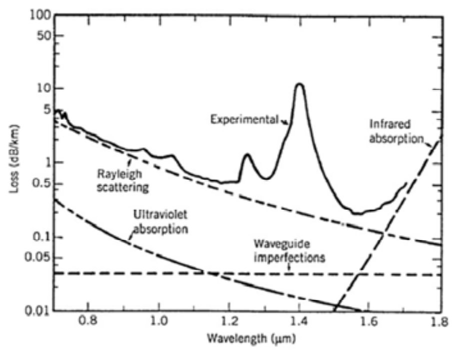
$$BW = BR/2$$

Fiber Losses

$$\frac{dP}{dz} = -\alpha P \quad (92)$$

$$P_{ies} = P_{in} e^{-\alpha L} \quad (93)$$

$$\alpha\left[dB/km\right] = -\frac{10}{L}\log\left(\frac{P_{ies}}{P_{in}}\right) \quad (94)$$



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Generally, changes in the average optical power P of a bit string propagating within an optical fiber is governed by Beer's law, (92), where α is the attenuation coefficient. This includes both material absorption and other losses sources such as core imperfections, Rayleigh scattering, ultraviolet absorption.

If Pin is the power released in the length L of the fiber, the output power will be given by the relationship (93).

Typically, the attenuation coefficient α is expressed in dB / km, using the relationship (94). In this case, it is called the fiber loss parameter.

Fiber losses depend on the wavelength of transmitted light.

The figure shows the wave length losses for a mono-mode fiber having a core diameter of 9.4 µm, Δ = 1.9 * 10 (-3) and a 1.1 µm cut-off wavelength. This fiber has losses of 0.2 dB / km in the wavelength region to 1.55 µm. This value is very close to the fundamental limit of 0.16 dB / km of a core with a core of silica.
A second minimum is 1.3 µm, where the losses are below 0.5 dB / km.
For lower wavelengths, the losses are considerably higher, exceeding 5 dB / km in the visible range.